Solution of Quadratic Equations by Graphical Methods

The general quadratic equation $ax^2 + bx + c = 0$ has real-valued coefficients a, b, c and $a \neq 0$. There are a number of methods of solution[1] in the document the solution of quadratic equations by graphical methods is explored. The graphs can be easily reproduced on the accompanying spreadsheet.

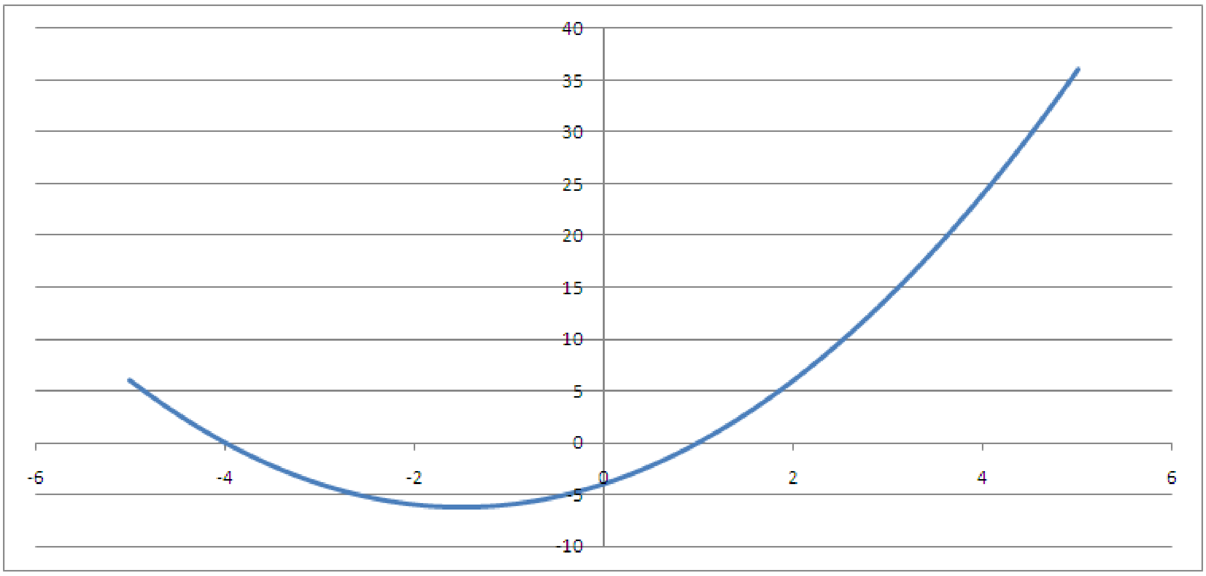
For the general quadratic equation $ax^2 + bx + c = 0$, we consider plotting the graph

$$y = ax^2 + bx + c.$$

When $y = 0$, or where the graph meets or crosses the x axis gives the solutions of the quadratic equation.

(a) Two distinct (real) solutions

For example the quadratic equation $y = x^2 + 3x - 4$ has the following graph



The curve – a parabola – crosses the x-axis when $x = -4$ or $x = 1$, the solutions of the quadratic equation $x^2 + 3x - 4 = 0$. This is typical of the case that we are most familiar with; two distinct (real) solutions.

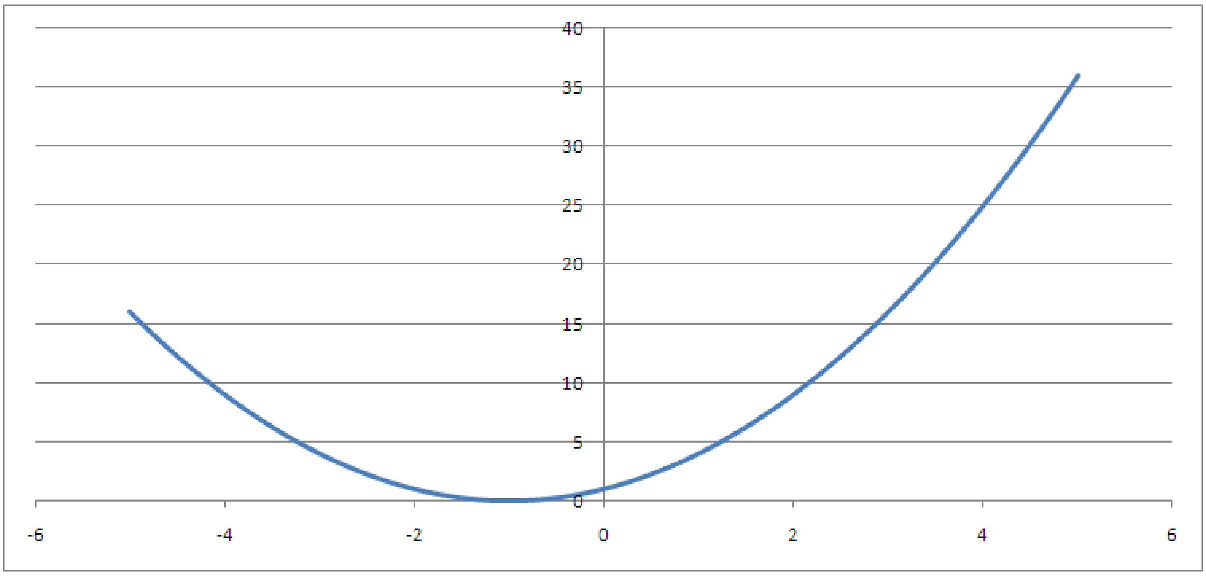
(a) One (repeated) solution

For example the quadratic equation $y = x^2 + 2x + 1$ has the following graph. The graph touches, rather than crossing, the x-axis; there is only one solution $x = -1$. If we factorise the equation

$$x^2 + 2x + 1 = (x + 1)(x + 1) = 0$$

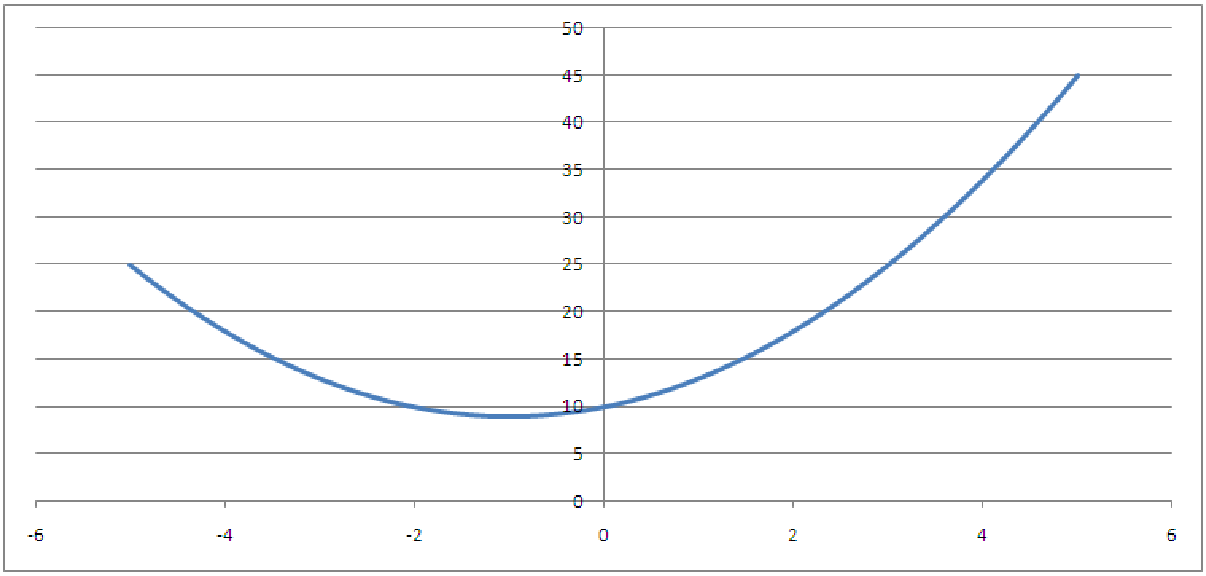
Showing the repeated solution $x = -1$.

[1] Solution of Quadratic Equations



(c) No (real) Solution

For example the quadratic equation $y = x^2 + 2x + 10$ does not have a (real) solution. The graph is as follows:



In the graph above, we can see that the curve does not cross the *x*-axis.

However, a quadratic equation always has solutions if we allow solutions that are complex numbers. The easiest way to determine the complex solutions is to use a formula and that will be considered in the next section.