

Solution of Quadratic Equations by Completing the Square

The general quadratic equation $ax^2 + bx + c = 0$ has real-valued coefficients a, b, c and $a \neq 0$. There are a number of methods of solution¹ in the document the solution of quadratic equations by completing the square is explored.

Let us demonstrate the method on the quadratic equation $x^2 + 3x - 4 = 0$.

The first step is to absorb the x term into the squared term. We can see that

$$\left(x + \frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}.$$

Hence the quadratic equation can be written as

$$0 = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 4.$$

It follows that

$$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}.$$

Hence

$$\left(x + \frac{3}{2}\right) = \pm \sqrt{\frac{25}{4}} = \pm \frac{5}{2}.$$

It follows that

$$x = -\frac{3}{2} \pm \frac{5}{2} = -4 \text{ or } 1.$$

¹ [Solution of Quadratic Equations](#)