

## Series and $\Sigma$ -notation

A series is the sum of a sequence<sup>1</sup>. A series may be finite (ie have a finite number of terms) or infinite (ie have an infinite number of terms). An infinite series is identified by the '+...' at the end.

For example

$$(a) 0 + 2 + 4 + 6 + 8 + 10 + 12$$

a finite series, each term is generated by adding two to the previous term

$$(b) 1 + 2 + 4 + 8 + 16 + 32 + 64 + \dots$$

an infinite sequence, each term is generated by multiplying the previous term by three

$$(c) 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

the reciprocals of 1,2,3,4,5,..

### Summation

The terms of a series (a sequence<sup>2</sup>) may be labelled by a general term or a symbol and index.

For example for a general term  $x_n$  the series is (d)  $x_1 + x_2 + x_3 + \dots$ .

Usually the index starts at 0 or 1.

The summation symbol  $\Sigma$  (Greek letter 'sigma') is may be used.

For example

$$(a) 2 + 4 + 6 + 8 + 10 + 12 = \sum_{i=1}^6 2i.$$

$$(b) 1 + 2 + 4 + 8 + 16 + 32 + 64 + \dots = \sum_{i=1}^{\infty} i^2$$

$$(c) 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \sum_{i=1}^{\infty} \frac{1}{i^2}$$

$$(d) x_0 + x_1 + x_2 + x_3 + \dots = \sum_{i=0}^{\infty} x_i$$

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<sup>1</sup> [Sequences](#)

### Sequence of partial sums

For the general sequence (d) above the sequence of partial sums is the sequence

$$S_1, S_2, S_3, \dots$$

where  $S_j = \sum_{i=1}^j x_i$ .

For example for (c), the sequence of partial sums is

$$1, 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{7}{8}, \dots$$

### Sum of a series

A finite series has a finite sum.

For example

$$(a) 0 + 2 + 4 + 6 + 8 + 10 + 12 = 42.$$

An infinite series can often have an infinite sum

For example

$$(b) 1 + 4 + 8 + 16 + 32 + 64 + \dots = \infty.$$

An infinite series can also have a finite sum which is the *limit*<sup>2</sup> of the sequence of partial sums.

$$(c) 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 2.$$

### Special Series

Special types of series include Arithmetic Series Geometric Series, McClaurin series, Taylor Series and Fouries Series.

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<sup>2</sup> [Sequences](#)