Geometric Series

A series[1] is a summation of a sequence of numbers. A geometric series or *geometric progression* is the sum of the terms of a *geometric sequence*[2]*.* The series may be finite or infinite.

For example

(a) $5 + 15 + 45 + 135 + 405$,

each term is generated by multiplying the previous term by three.

(b) 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125... ,

each term is generated by halving the previous term.

In general the sequences have the *general term* ar^n for fixed values of *a* and *r* and with *n*=0,1,2,3,4...

For example for

(a) $5 + 15 + 45 + 135 + 405$.

$$a = 5, r = 3\ ,$$

(b) 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125... ..,

$$a = 8, r = 0.5\ ,$$

are geometric series; (a) is a finite geometric series since it has a finite number of terms, whereas (b) is an infinite geometric series (the ... indicates that the terms continue infinitely).

Finite Geometric Series

For a finite geometric series with the general term ar^n , with the first term *a* and with the final term ar^N, the sum of the series is given by the formula

$$\text{SUM} = a\left(\frac{1-r^N}{1-r}\right)$$

[1] Series
[2] Geometric Sequences

For example for the series

$$5 + 15 + 45 + 135 + 405,$$

a=5, r=3, N=4, Hence the sum is $5\left(\frac{1-3^5}{1-3}\right) = 5\left(\frac{-242}{-2}\right) = 605$.

Infinite Geometric Series

The series (b) 8 + 4 + 2 + 1 + 0.5 + 0.25 + 0.125... is an infinite geometric sequence. The sum of an infinite geometric series is given by the formula

$$\text{SUM} = \frac{a}{1-r} \quad (-1<r<1).$$

Hence for example (b), SUM = $\frac{8}{1-0.5} = \frac{8}{0,5} = 16.$

Note that critically, the terms must reduce successively (-1<r<1) in magnitude for the sum of a geometric series to have a finite sum.