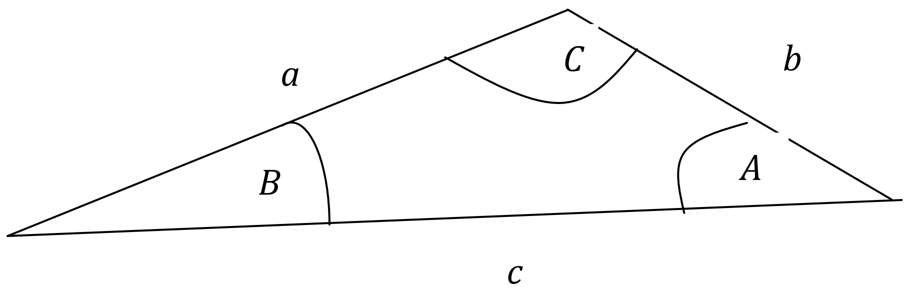
Sine and Cosine Rules

The sine and cosine rules are equations that relate the lengths of sides and angles of triangle. Let the triangle have sides of length a, b and c, with the angles opposite each side having angles of A, B and C.



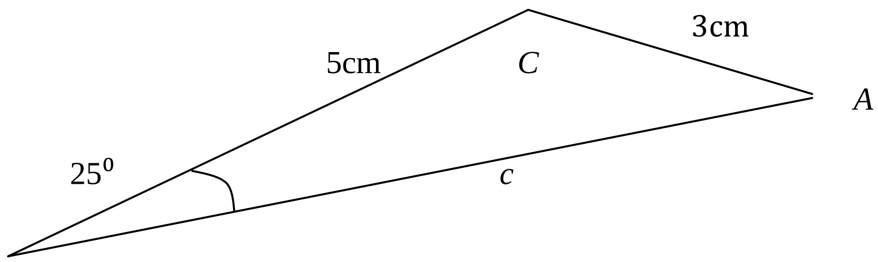
Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To use the sine rule either two sides with one of their opposite angles are given or two angles and one of their opposite sides.

Example

Determine the angles A and C and the length of side c.



Using the sine rule it follows that $\frac{3}{\sin 25°} = \frac{5}{\sin A} = 7.099$.

Hence $A = 44.78°$. Since the internal angles of a triangle have the sum of $180°$, then $C = 110.22°$.

Using the sine rule it follows that $\frac{c}{\sin 110^0} = 7.099$.

Hence $c = 6.67$cm.

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\ cos\ A$$

Alternatively:

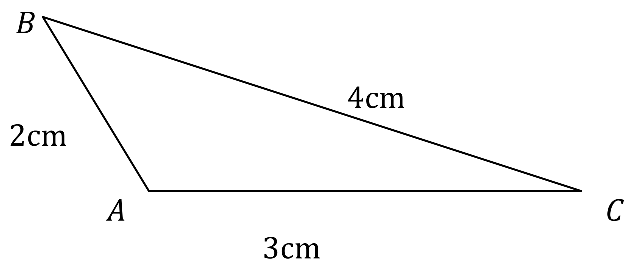
$$b^2 = a^2 + c^2 - 2ac\ cos\ B$$

$$c^2 = a^2 + b^2 - 2ab\ cos\ C$$

To use the rule two either all three side lengths need to be known, or two sides and the angle between them.

Example

A triangle has sides of length 2cm, 3cm and 4cm. Find the internal angles.



We have a=4cm, b=3cm, c=2cm. Any of the formulae above can be used to progress with the solution, so we will work from the first one:

$$4^2 = 3^2 + 2^2 - 3 \times 2 \times \cos A\ .$$

Hence

$$16 = 9 + 4 - 6 \cos A$$

Hence

$$\cos A = -0.5 \text{ and } A{=}120^0 \text{ (or } \tfrac{2}{3}\pi\text{).}$$

To find another internal angle we may use either the sine or cosine rule. Using the sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} \text{ gives } \frac{4}{\sin 120^0} = \frac{3}{\sin B}\ .$$

Hence

$$\sin B = \tfrac{3}{4} \sin 120° = 0.650 \text{ and } B{=}\ 40.51°.$$

Since all the internal angles sum to 180° then C=19.49°.