# Sine, Cosine and Tangent of Sum

In this document two important identities in trigonometry; those that derive the sine and cosine of a sum of angles. For sine we have the following identity:

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)\,. \tag{1}$$

For cosine we have the identity

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B). \tag{2}$$

For tangent we have the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \tag{3}$$

Example

Find $\sin(70°)$, given that $\sin(30°) = 0.5$, $\sin(40°) = 0.6428$, $\cos(30°) = 0.8660$ and $\cos(40°) = 0.7660$.

From equation (1) above we can state:

$$\sin(70°) = \sin(30°)\cos(40°) + \sin(40°)\cos(30°)$$

Continuing to work to four decimal places, this gives

$$\sin\,(70°) = 0.5 * 0.7660 + 0.8660 * 0.6428 = 0.3830 + 0.5567 = 0.9397$$

Example

Find $\cos\,(70°)$ given that $\sin(30°) = 0.5$, $\sin(40°) = 0.6428$, $\cos(30°) = 0.8660$ and $\cos(40°) = 0.7660$.

From equation (2) above we can state:

$$\cos(70°) = \cos(30°)\cos(40°) - \sin(30°)\sin(40°)$$

Continuing to work to four decimal places, this gives

$$\cos(70°) = 0.8660 * 0.7660 - 0.5 * 0.6428 = 0.6634 - 0.3214 = 0.3420$$

Example

Find tan (70°) given that tan (30°) = 0.5774, tan(40°) = 0.8391.

From equation (3) above we can state:

$$\tan(70°) = \frac{\tan 30° + \tan 40°}{1 - \tan 30° \tan 40°} .$$

Hence

$$\tan(70°) = \frac{0.5774 + 0.8391}{1 - 0.5774 \times 0.8391} = \frac{1.4165}{0.5155} = 2.7475 ,$$

working to four decimal places.

## Sine, Cosine and Tangent of double angles

For double angles, equations (1)-(3) simplify to equations (4)-(6):

$$\sin(2A) = 2 \sin A \cos A , \tag{4}$$

$$\cos(2A) = \cos^2 A - \sin^2 A , \tag{5}$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A} . \tag{6}$$

As a result of Pythagoras's Theorem[1], $\sin^2 A + \cos^2 A = 1$. Hence equation (5) can be re-written as

$$\cos(2A) = 1 - 2 \sin^2 A \tag{5a}$$

or

$$\cos(2A) = 2\cos^2 A - 1 . \tag{5b}$$

[1] Trigonometry