Matrix Arithmetic

We can apply elementary arithmetic operations to matrices. The matrix operations of addition, subtraction and multiplication are defined. The operation of division is analogous to multiplication by the inverse of a matrix and this is treated in a separate document.

Matrix Addition

In order to add two matrices we simply add each of the individual elements.

For example $\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 2+1 & -1+2 \\ 1+(-2) & 3+1 \\ 4+0 & -2+5 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -1 & 4 \\ 4 & 3 \end{pmatrix}.$

This only makes sense if the matrices have the same dimensions.

Matrix Subtraction

In order to add two matrices we simply add each of the individual elements.

For example $\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 2-1 & -1-2 \\ 1-(-2) & 3-1 \\ 4-0 & -2-5 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 3 & 2 \\ 4 & -7 \end{pmatrix}.$

As with addition, this only makes sense if the matrices have the same dimensions.

Matrix Multiplication

The result of multiplying two matrices is another matrix for which the ij^{th} element is obtained by multiplying each element of the i^{th} row of the first matrix by each element of the j^{th} row of the second matrix and summing the result.

For example

$$\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 2\times 1+(-1)\times(-2) & 2\times 3+(-1)\times 4 \\ 1\times 1+3\times(-2) & 1\times 3+3\times 4 \\ 4\times 1+(-2)\times(-2) & 4\times 3+(-2)\times 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ -5 & 15 \\ 8 & 4 \end{pmatrix}.$$

Two matrices can only be multiplied if the number of columns of the first matrix is equal to the number of rows of the second matrix. The resultant matrix has the same number of rows as the first matrix and the same number of columns as the second matrix.

The square of a matrix can be found by multiplying it by itself.
For example

$$\begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}^2 = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 15 \\ -10 & 10 \end{pmatrix}.$$