Inverse of a 2x2 Matrix

In this document we show how we can find the inverse of a 2x2 matrix.

Determinant

The determinant is an important property of a matrix. For a 2x2 matrix

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix},$$

the determinant is defined as follows:

$$\det(A)=a_{11}a_{22}-a_{21}a_{12}.$$

Example 1

For the matrix $A=\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, det(A)=2x2-3x1=4-3=1.

Inverse

The inverse of a square matrix A is denoted A^{-1}. It has the following property:

$$A\,A^{-1}=A^{-1}A=I.$$

For a 2x2 matrix

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix},$$

the inverse is defined as

$$A^{-1}=\frac{1}{\det(A)}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}.$$

Example 2

For the matrix $A=\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, $A^{-1}=\frac{1}{1}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}=\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$.

A matrix does not necessarily have an inverse. If the determinant is zero then clearly the inverse cannot be defined. A matrix without an inverse is said to be singular.

<u>Example 3</u>

For the matrix A= $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$, det(A)=2x2-4x1=0. It has no inverse.

Spreadsheet

The accompanying spreadsheet can compute the inverse of a 2x2 matrix.

			Matrix inverse	www.mathematics.me.uk
A=	2	1		change values in yellow background
	3	2		
determinant=		1		
A^(-1)=	2	-1		the inverse
	-3	2		
A*A(-1)=	1	0		matrix multiplied by its inverse gives the identity matrix
	0	1		