Laplace Transforms

Definition

For any function $f(t)$ its Laplace transform is defined to be

$$F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt.$$

Table of Common Laplace Transforms

Number	$f(t)$	$F(s)$
0	$\delta(t)$	1
1	1	$\frac{1}{s}$
2	k	$\frac{k}{s}$
3	e^{-at}	$\frac{1}{s+a}$
4	$\sin(at)$	$\frac{a}{s^2+a^2}$
5	$\cos(at)$	$\frac{s}{s^2+a^2}$
6	t	$\frac{1}{s^2}$
7	t^n (n is a positive integer)	$\frac{n!}{s^{n+1}}$
8	$\cosh(at)$	$\frac{s}{s^2-a^2}$
9	$\sinh(at)$	$\frac{a}{s^2-a^2}$
10	$e^{-at}t^n$	$\frac{n!}{(s+a)^{n+1}}$
11	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
12	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
13	$e^{-at}\cosh(\omega t)$	$\frac{s+a}{(s+a)^2-\omega^2}$
14	$e^{-at}\sinh(\omega t)$	$\frac{\omega}{(s+a)^2-\omega^2}$

The Laplace Transform of Derivatives

The Laplace transform of the derivatives of a function $f(t)$ are defined as follows:

$$L\left\{\frac{df}{dt}\right\} = sF(s) - f(0),$$

$$L\left\{\frac{d^2f}{dt^2}\right\} = s^2F(s) - sf(0) - f'(0),$$

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$$L\left\{\frac{d^nf}{dt^n}\right\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0).$$

Other properties

$$L\{f(t) + g(t)\} = L\{f(t)\} + L\{g(t)\}$$