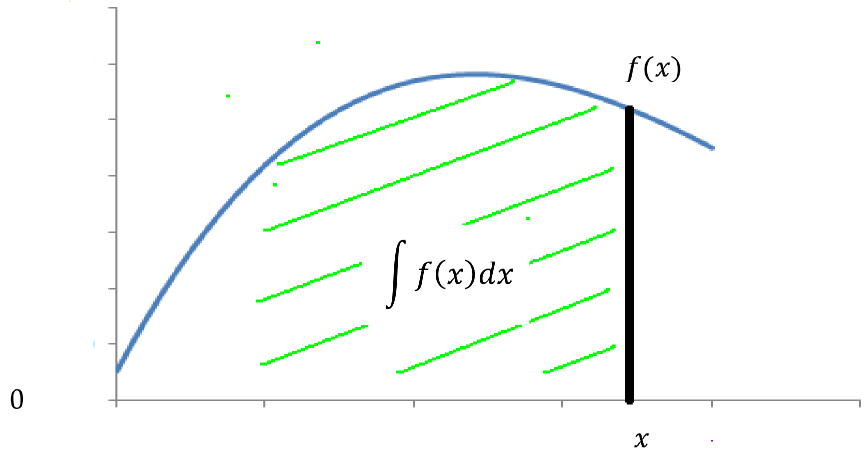
Integration

Integration measures the accumulation of the value of a function. The symbol for integration is "$\int$ ", and it can be usefully viewed as a stretched out "S" and meaning *sum*. The *integral* of a function $f(x)$ is expressed mathematically as "$\int f(x)dx$". Integration is the inverse of differentiation[1].

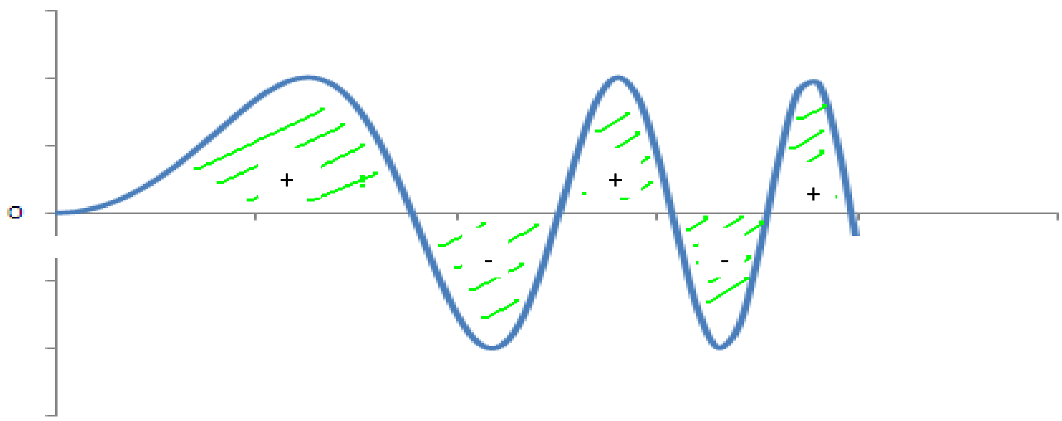
Indefinite Integral

The integral of a function $f(x)$, $\int f(x)dx$ is the accumulation of the value of $f(x)$ with x. It can be viewed as the value of the area below the graph of $f(x)$ as a function of x. In the following illustration, $\int f(x)dx$ can be represented by the area under the graph of f up to x.



Note that the position in which we start to measure the area is unspecified; the integral is said to be indefinite. However, the form of the accumulation function can be determined and indefiniteness of the integral can be accommodated by including an arbitrary constant of integration.

Note that when the graph is negative then the area is interpreted as negative area, as illustrated in the following graph



[1] Differentiation

Indefinite Integrals of Standard Functions[2]

$f(x)$	$\int f(x)dx$
$x^n \ (n \neq -1)$	$\frac{1}{n+1}x^{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln x + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
e^x	$e^x + c$
$\ln x$	$x \ln x - x + c$
$\sinh x$	$\cosh x + c$
$\cosh x$	$\sinh x + c$
$\frac{1}{(1+x^2)}$	$\tan^{-1} x + c$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + c$
$\frac{-1}{\sqrt{1-x^2}}$	$\cos^{-1} x + c$
$\frac{1}{\sqrt{1+x^2}}$	$\sinh^{-1} x + c$
$\frac{-1}{\sqrt{1+x^2}}$	$\cosh^{-1} x + c$
$\frac{1}{(1-x^2)}$ for $\lvert x \rvert < 1$	$\tanh^{-1} x + c$
$\frac{1}{(1-x^2)}$ for $\lvert x \rvert > 1$	$\coth^{-1} x + c$

[2] Standard Mathematical Functions: Windows Scientific Calculator

Rules for Integration

Integral of a function with a constant factor

$$\int a\, u(x) dx = a \int u(x)\, dx$$

For example

$$\int 3\, x^2 dx = 3 \int x^2\, dx = 3\, \frac{x^3}{3} + c = x^3 + c$$

Integral of a sum

$$\int \big(u(x) + v(x)\big) dx = \int u(x) dx + \int v(x)\, dx$$

For example

$$\int (\cos x + \ln x) dx = \int \cos x\, dx + \int \ln x\, dx = \sin x + x \ln x - x + c$$

Integral of a chain rule derivative

$$\int u'\big(v(x)\big) v'(x) dx = u\big(v(x)\big) + c$$

For example

$$\int \cos(\sin x) \cos x\, dx = \sin(\sin x)) + c$$

Integration by parts

$$\int v(x) \frac{du(x)}{dx} dx = u(x)v(x) - \int u(x) \frac{dv(x)}{dx} dx$$

One of the uses of this equation is when $\frac{dv(x)}{dx}$ is a simpler function than $v(x)$.

For example for the integral

$$\int x \cos x\, dx$$

it is noted that if we set $v(x) = x$, then - on the other side of the integration by parts formula - $\frac{dv(x)}{dx} = 1$; hence simplifying the expression. It therefore follows that $\frac{du(x)}{dx} = \cos x$ and hence that $u(x) = \sin\ x$.

$$\int x \cos x\, dx = (\sin x)x - \int (\sin x\ )\ 1\, dx = x \sin x + \cos x + c$$

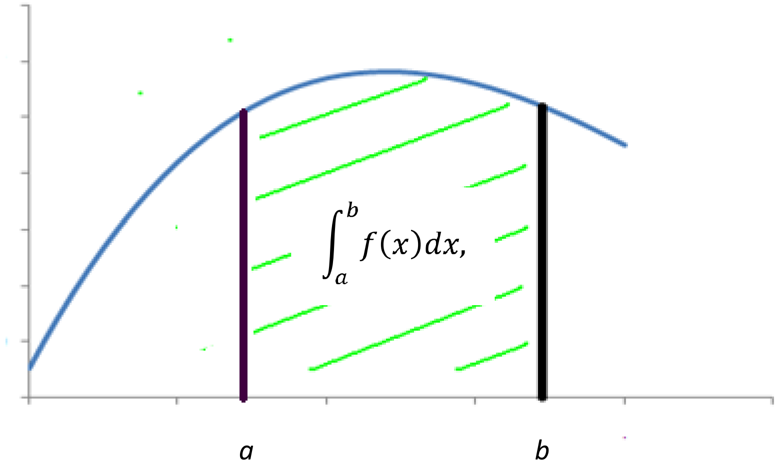
Definite Integrals

A definite integral sets limits on the domain of integration. The integral sign is written

$$\int_a^b f(x)dx,$$

and it is evaluated as a numeric value, rather than a function.

The value of the integral is equal to the area illustrated in the following diagram:



Example

$$\int_1^2 x^3 dx = \left[\frac{x^4}{4}\right]_1^2 = \left(\frac{2^4}{4} - \frac{1^4}{4}\right) = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}.$$