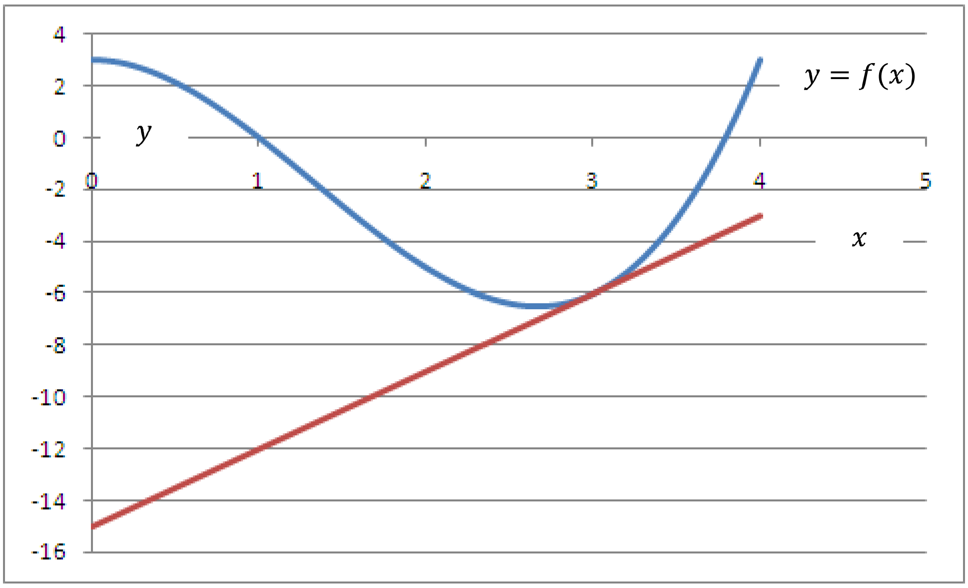
Differentiation

Differentiation is to do with finding the rate of change. If we consider a general function $y = f(x)$ then differentiating that function is equivalent to finding its derivative. The derivative function is denoted $\frac{dy}{dx}$ or $f'(x)$. Differentiation is the inverse of integration[1]

In order to illustrate differentiation, consider first the curved graph below of $y = f(x)$.

At any point we can draw a tangent to the curved line and this tangent changes at every point along the curve. In the graph below the tangent at $x = 3$ is shown.



The gradient of the tangent at every point x is equal to the derivative of $f(x)$.

Differentiation is the reverse of the process of integration.

[1] Integration

Derivatives of Standard Functions[2]

$f(x)$	$f'(x)$
$x^n \ (n \neq 0)$	nx^{n-1}
e^x	e^x
$\ln x$	$1/x$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2\, x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\tan^{-1} x$	$\dfrac{1}{(1+x^2)}$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\dfrac{-1}{\sqrt{1-x^2}}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\text{sech}^2\, x$
$\coth x$	$-\text{cosech}^2\, x$
$\text{sech}\, x$	$\text{sech}\, x \tanh x$
$\text{cosech}\, x$	$-\text{cosech}\, x \coth x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{-1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$ for $\lvert x \rvert < 1$	$\dfrac{1}{(1-x^2)}$
$\coth^{-1} x$ for $\lvert x \rvert > 1$	$\dfrac{1}{(1-x^2)}$

2 Standard Mathematical Functions: Windows Scientific Calculator

Rules for Differentiation

Derivative of a function with a constant factor

$$\frac{d}{dx}\{c\,u(x)\} = c\frac{d}{dx}u(x) = c\,u'(x)$$

For example

$$\frac{d}{dx}\{3\sin x\} = 3\frac{d}{dx}\sin x = 3\cos x$$

Derivative of a sum

$$\frac{d}{dx}\{\,u(x) + v(x)\} = \frac{d}{dx}u(x) + \frac{d}{dx}v(x) = u'(x) + v'(x)$$

For example

$$\frac{d}{dx}\{\ln x + \cos x\} = \frac{d}{dx}\ln x + \frac{d}{dx}\cos x = \frac{1}{x} - \sin x$$

Derivative of a product

$$\frac{d}{dx}\{\,u(x)v(x)\} = u(x)v'(x) + u'(x)v(x)$$

For example

$$\frac{d}{dx}\{\ln x \sin x\} = \ln x\frac{d}{dx}(\sin x) + \frac{d}{dx}(\ln x)\sin x = \ln x\cos x + \frac{1}{x}\sin x$$

Derivative of a quotient

$$\frac{d}{dx}\left\{\frac{u(x)}{v(x)}\right\} = \frac{v(x)u'(x) - u(x)v'(x)}{v^2(x)}$$

For example

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}(\cos x)}{\cos^2 x} = \frac{\cos x\cos x - \sin x\,(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Derivative of a function of a function

$$\frac{d}{dx}u\big(v(x)\big) = v'(x)u'(v(x))$$

For example

$$\frac{d}{dx}(\sin(\ln x)) = \frac{d}{dx}(\ln x)\cos(\ln x) = \frac{1}{x}\cos(\ln x))$$